Noise and Noise Meter

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Content

Abstract

In order to make use of analysis and low noise design techniques one requires a knowledge of the noise performance of circuit elements, circuits and complete systems in order to determine whether they meet the specifications or not. Also to judge the effort of noise decreasing procedures in circuits one need to be able to measure the noise level.
If high precision noise measurement is required one can employ a noise meter.

1 Introduction

There are two commonly used definitions of electronic noise: Noise is either a random fluctuation in potential difference or current resulting from the random movement of charge carriers, or it is an unwanted signal tending to obscure or interfere with a required signal.
The second definition is broader and includes the random noise but it also describes interference – that is, the addition to the "required" signal of other signals which may or may not be random.
An example of a non-random or deterministic noise signal is mains (line) hum in an audio system. This is a 50 or 60 Hertz signal (together with harmonics) introduced into the audio signal path by means of magnetic induction from a transformer or capacitate coupling between the power supply and audio signal leads.

1.1 Noise categories

Electronic noise may be conveniently divided into two categories determined by the noise source. In the first category the noise source is external to the elements of the circuit under consideration. In the second category the noise is generated within the circuit elements.
Noise in the first category (external noise source) is generated in some external source and is coupled into the circuit of interest. This can be electronic radiation from transmitters or brush motors, the alternating magnetic field from a transformer and many more.
Noise in the second category is caused by the random movement of charge carriers within conductors. Thermal noise, which is present even in the absence of current flow, results from the thermally induced random motion of charge carriers and is dependent of temperature.

1.2 Noise Forms

Shot noise:
Shot noise is the random component of current flowing across a potential barrier and results from the random crossing of the potential barrier by the charge carriers. Its spectral density is constant and follows the Gaussian probability distribution.

1/f noise:
At low frequencies the shot noise in semiconductures is overshadowed by another type of noise, so called low frequency noise or 1/f noise. The law of variation for the spectral density of this noise is expressed approximately by

$$w(f) = \frac{K}{f^{v}} \qquad (1)$$

where the value of v ranges from 0.8 to 1.5.

White noise:
A noise source is said to be white if the available mean noise power per unit bandwidth is a constant at all frequencies.

Gaussian Noise:
The amplitude values of Gaussian noise follows – as the name suggest – the Gaussian probability distribution. Its significance with respect to noise lies in the fact that a noise source is often a composite sum of large number of independent processes and hence follows a gaussian distribution.

2 Noise Meter [1 p. 145-163]

When it is important to measure noise accurately a meter is required.
The meter consists of an amplifier, a rectifier, an averager and a DC meter.

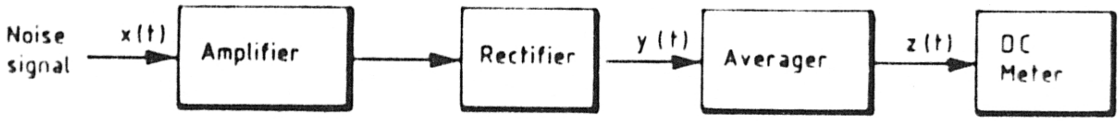

Figure 2.1: Noise meter [1]

Very important in respect to complexity and cost of the equipment is the required accuracy of the noise measurement. As in other measurement tasks one should avert from using more accuracy than appropriate to the design problem under consideration.

2.1 Description of the Components

2.1.1 Amplifier

The amplifier has the task to amplify the noise signal to a level suitable for the rectifier and DC meter used. Therefore it should have sufficient gain not to cut the input values, except for the occasional extreme values of the noise. One should remember that a sinusoidal wave is contained within only $\pm\sqrt{2}\times rms$, whereas there are a significant number of excursions of a Gaussian waveform up to $4\times rms$. Also its own noise level should be small compared with the noise to be measured.

2.1.2 Rectifier

Preferably the measurement system should display an indication of true rms value – giving an accurate measurement of rms voltage, whatever the input waveform shape is. In this type of measurement system the amplified noise signal is passed to a squaring circuit or square law rectifier. The output of the averager is then the mean square voltage. If one uses an analogue or pointer meter for displaying the meter scale has to be calibrated to the square root of the applied voltage, otherwise a analogue or digital square rooting circuit it used between averager and meter (see figure 2.1), before displaying on a linear scale meter.

Some systems use a diode rectifier to measure alternating voltages. These readings depend on the input wave form and are not true rms. For measurement aberrations of noise in a system calibrated for sinusoid formed inputs, see chapter 2.3: Aberrations for a sinusoid calibrated rectifier.

2.1.3 Averager

The output of the squarer or diode rectifier is a random quantity and requires smoothing or averaging before display.

This can be running average, using a low pass filter with a impulse response of $h_L(t)$:

$$z(t) = y(t) * h_L(t) \qquad (2)$$

where $y(t)$ is the output signal of the rectifier and $z(t)$ the output of the averager.
Another way is to integrate the signal over a period of time τ :

$$z(t) = \frac{1}{\tau}\int_{t-\tau}^{t} y(t)dt \qquad (3)$$

In practice the integration is usually carried out on consecutive signal segments of the duration τ , rather than continuously, leading to averages updated at intervals $n\tau$ *(n=1, 2, 3, …)*:

$$z(n\tau) = \frac{1}{\tau}\int_{(n-1)\tau}^{n\tau} y(t)dt \qquad (4)$$

With a moving coil meter, the low pass filtration may be provided by mechanical damping of the meter movement.

For discussion about aberrations caused by the averager, see chapter 2.4: Deviation of the reading in respect to the averager.

2.2 measurement bandwidth

One can distinguish broad bandwidth measurement and spot frequency measurement, depending on the desired result. The needed type of measurement can be achieved by adapting or exchanging the amplifier used.

2.2.1 broadband measurement

The bandwidth of the amplifier should be greater than that of the noise to be measured. It is important to note that it may need to be very significantly greater, depending on the shape of the noise spectrum and that of the amplifier frequency response.
For example, if the noise is white noise, low-pass filtered by a single pole CR network and the amplifier is similarly high frequency limited by a single pole filter, then the spectrum of the noise at the input of the amplifier is

$$S_n(f) = \frac{S_n(0)}{1+(f/f_n)^2} \qquad (5)$$

and at the output:

$$S_{on}(f) = S_n(f)G_a(f) \qquad (6)$$

with:

$$G_a(f) = G_a(0)(1+(f/f_a)^2)^{-1} \qquad (7)$$

as the frequency-dependent, normalised power gain of the amplifier, where f_n and f_a are the 3dB cut-off frequencies of the input noise and amplifier response respectively and $S_n(0)$ is the spectral density of the input noise at low frequencies $(f << f_n)$.

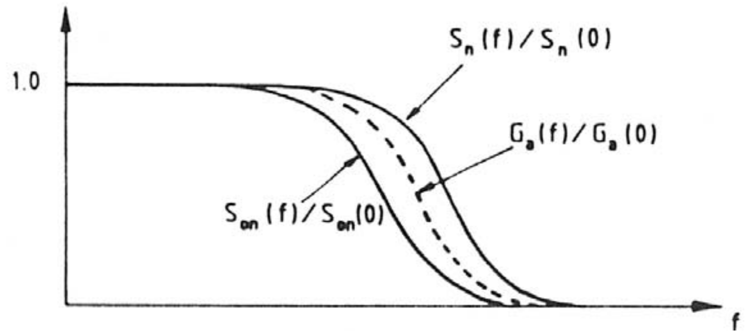

Figure 2.2: Spectral densities of noise at input and output of the amplifier together with the normalised power gain frequency response

The actual normalised noise power is:

$$e_n^2 = \int_0^\infty S_n(f)df = \frac{\pi}{2} S_n(0) f_n \qquad (8)$$

and the measured normalised noise power:

$$e_{On}^2 = \int_0^\infty \frac{S_{On}(f)df}{G_a(0)} = \frac{\pi}{2(1+1/\beta)} S_n(0) f_n \qquad (9)$$

where is $\beta = f_a / f_n$.

The fractional measurement error can be put as:

$$\varepsilon_a = \frac{e_{On}^2 - e_N^2}{e_N^2} = -\frac{1}{1+\beta} \qquad (10)$$

and is shown for a range of β in table 2.3.

Ratio of measurement to noise bandwidths β	*Measurement error* ε_a *(%)*
1	−50.0
2	−33.3
5	−16.7
10	−9.1
20	−4.8
50	−2.0
100	−1.0

Table 2.3: Error in measured normalised noise power against ratios of amplifier-to-noise bandwidth $\beta = f_a / f_n$

It is clear that a measurement bandwidth very much greater than the noise bandwidth is required for accurate noise power measurement if the bandwidths are limited that way.

2.2.2 Spot frequency measurement

When making spot frequency measurements the amplifier includes a narrow band filter. If more than a few spot frequency measurements are required, it is easier to use a spectrum analyzer [1 p. 158].
The width of the band filter is important: The term ‘spot frequency’ implies that the noise spectral density is measured at one frequency. In practice one have to measure the noise power within a narrow frequency band Δf in order to calculate spectral density. The time required to complete a measurement to the needed accuracy is inversely proportional to this bandwidth. With a narrow bandwidth of a few Hertz and a reasonable accuracy requirement this time can amount several minutes, so there is good reason to use a measurement bandwidth as wide as possible.

The deterministic error in spectral density measurement is determined by the change in spectral density of the noise over the bandwidth of the measurement filter. Considering the measurement of spectral density at frequency f_0 using a filter and amplifier combination having response $G_m(f)$ with centre frequency f_0, then the measured mean square voltage is:

$$e_M^2 = \int_0^\infty S_x(f) G_m(f) df \qquad (11)$$

where $S_x(f)$ is the spectral density of the input noise. If $G_m(f)$ is sufficiently narrow, that $S_x(f)$ can be considered constant and equal to $S_x(f_0)$ over the extent of $G_m(f)$, then one can write:

$$e_M^2 = S_x(f_0) \int_0^\infty G_m(f) df = S_x(f_0) G_{po} B_m \qquad (12)$$

with G_{po} as the gain at frequency f_0 and the measurement bandwidth B_m:

$$B_m = \frac{\int_0^\infty G_m(f) df}{G_{po}} \qquad (13)$$

By using equation (12) one can express the spectral density at $f = f_0$ by:

$$S_x(f_0) = \frac{e_M^2}{G_{po} B_m} \qquad (14)$$

The effect of small variations of $S_x(f)$ over the measurement bandwidth can be analysed by expressing $S_x(f)$, within the extent of $G_m(f)$, as a Taylor series expansion about its value at $f = f_0$ and considering the first three terms:

$$S_x(f) \approx S_x(f_0) + S_x^{'}(f_0)(f - f_0) + \frac{1}{2} S_x^{''}(f_0)(f - f_0)^2 \qquad (15)$$

The measured mean square voltage from equation (11) is now:

$$e_M^2 = S_x(f_0) G_{po} B_m + S_x^{'}(f_0) \int_0^\infty G_m(f)(f - f_0) df + \frac{1}{2} S_x^{''}(f_0) \int_0^\infty G_m(f)(f - f_0)^2 \qquad (16)$$

Normally the measurement filter response is symmetrical about f_0, so the integral of $G_m(f)(f - f_0)$ is zero. Thus, if $G_m(f)$ is symmetrical, a linear variation of $S_x(f)$ will not give rise to an error in measurement (all higher derivations from $S_x(f)$ than one will be zero).

But if there is a significant curvature of $S_x(f)$, then $S_x^{''}(f_0)$ is not neglible anymore.

For a rectangular $G_m(f)$ one can derive that for $1/f$ - noise the fractional error is:

$$\varepsilon_m = \frac{B_m^2}{12 f_0^2} \qquad (17)$$

which suggests that a fairly high fractional measurement bandwidth B_m / f_0 may be used in this case. The complete derivation can be found in [1 p. 152].

For example if $B_m / f_0 = 0.5$, the fractional error is only about 2 per cent.

2.3 Aberrations for a sinusoid calibrated rectifier

Some noise meters use a diode rectifier, which readings depends on the input waveforms because they are not true rms indicating. These measurement systems will normally be calibrated for sinusoid formed inputs.

The mean value of a full wave rectified sine wave is $(2\sqrt{2}/\pi) \times rms$ and this reading is multiplied by $\pi/(2\sqrt{2})$ before display. However, the mean of a full wave rectified Gaussian noise signal is $\sqrt{(2/\pi)} \times rms$, so that the meter calibrated to indicate correctly the rms level of a sine wave gives a reading of:

$$\pi/(2\sqrt{2}) \times \sqrt{2/\pi} \times rms = 0.886 \times rms \qquad (18)$$

with Gaussian noise.

If such a meter is used for the measurement of Gaussian noise, the reading have to be multiplied by $1/0.886 = 1.13$. Of course, for mixed noise it will not give the correct result. Note that the average output of a half-wave rectifier is half that of a full wave for both sine and Gaussian, so the same correction factor may be used in this case.

2.4 Derivation of the reading in respect to the averager

The averaged square or rectified signal has a mean value and a random variation about this mean. To achieve a high accuracy one have to minimize this random variation before displaying.
The degree of random variation is dependent on the signal bandwidth and the averaging time and thus is also the accuracy of any single reading.

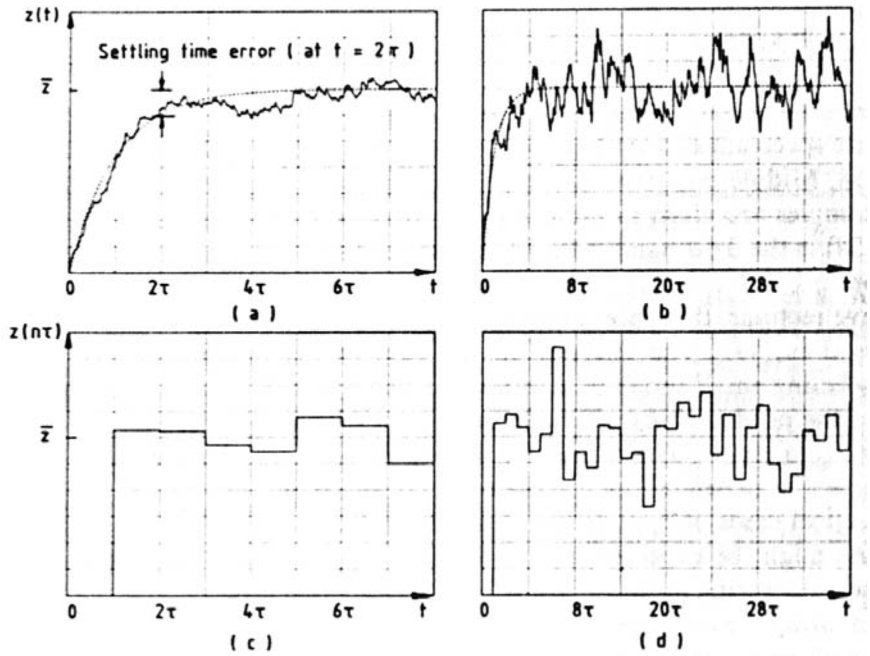

Figure 2.4: Averager output: (a, b) single pole CR low pass filter, (a) large τ , (b) small τ $(\tau = CR)$
(c, d) integrator, (c) large τ , (d) small τ (τ integration time) [1]

In general, for Gaussian noise the rms deviation of the reading from the mean is given by:

$$\delta_r = \frac{\text{rms deviation of meter reading from mean}}{\text{mean meter reading}} = \frac{1}{k\sqrt{B\tau}} \qquad (19)$$

where *B* is the bandwidth of the noise signal when making a broadband measurement, or the measurement bandwidth $(B = \Delta f)$ when making spot frequency measurements (see chapter 2.2), and τ is the averaging time for integration or the time constant for a low pass filter. The value of *k* depends on the spectrum shape, the definition of *B*, the type of averaging and whether a squarer or diode rectifier is used. One can simply write:

$$k = k_1 k_2 k_3 \qquad (20)$$

where the value of k_1 is determined by the type of rectifier, k_2 by the noise spectrum and k_3 by the type of averaging. Examples are to find in table 2.5.

Type of rectifier	k_1
Square law	1
Square law (square root taken after averager – true rms reading)	2
Full or half wave diode	2
Input Spectrum	k_2
White noise filtered by: rectangular bandpass or low-pass filter	1
single *CR* low-pass filter	$\pi^{1/2}$
Gaussian bandpass filter	$\left(\frac{\pi}{2\ln 2}\right)^{1/4}$
$1/f$ noise filtered by rectangular bandpass filter with passband from f_1 to f_2	$\frac{(f_1 f_2)^{1/2}\ln(f_2/f_1)}{f_2 - f_1}$
	$= 1.0$ $f_2/f_1 = 1.1$
	$= 0.98$ $f_2/f_1 = 2.0$
	$= 0.81$ $f_2/f_1 = 10$
	$= 0.47$ $f_2/f_1 = 100$
Averaging	k_3
Single *CR* low-pass filter ($\tau = CR$)	$2^{1/2}$
Integrator	1

Table 2.5: Examples for k_1, k_2, k_3 (good approximations) [1]

For these values of k_2 in Table 1.2 the *B* in equation (19) is the 3dB bandwidth in each case. It has been assumed that $B t_a >> 1$ and that in the case of a half wave rectifier, there are no signal frequency components of the order of, or less than, $1/\tau$. The analysis, in the case of a diode rectifier, involves neglecting small terms in a series expansion of the post-rectifier spectrum, and the product of $k_1 k_2$ will be in error by a few per cent, depending on the input spectrum shape. In the case of filtered $1/f$ noise, the expression for k_2 is frequency dependent since the spectral shape is not constant. For most practical cases it may be put to unity.
As might be expected, the values of k_1 indicate that the fractional error when measuring mean square voltage (normalised power) is twice that when measuring the corresponding root mean square. The apparent slight advantage of using a running average CR filter as opposed to an integrator is illusory, because one have to wait for the output of the filter to reach a stationary condition (see figure 2.4) before taking a measurement, whereas the output of the integrator is available after the integration period.

It is instructive to consider the integration time required for typical situations. From equation (19) one can derive:

$$\tau = \frac{1}{k^2 \delta_r^2 B} \qquad (21)$$

For the case of rectangular bandwidth noise, an rms indicating meter and a linear integrator, this is:

$$\tau = \frac{1}{4 \delta_r^2 B} \qquad (22)$$

For $B = 10\,kHz$ (wideband audio frequency noise) and a required rms error of 5 per cent one get an integration time of $\tau = 10\,ms$.
But if one is making a spot frequency measurement at low frequencies with a measurement bandwidth of $10\,Hz$, then the same rms error requires an integration time of $10\,s$. An error of 1 per cent would require an integration time of $250\,s$. If a running average CR filter is used, then these values (the time constant $\tau = CR$) are halved, but the settling time must be taken into account. There will be an additional error of 5 per cent after a time of $3CR$, or 1.8 per cent after a time of $4CR$.
These figures illustrate one of the problems of making low bandwidth noise measurements.

3 summary

For high precision noise measurement of broadband or few spot frequencies the noise meter is very useful.
It consists of an amplifier, a rectifier, an averager and a DC meter.
One can distinguish broad bandwidth measurement and spot frequency measurement, depending on the desired result, each with its own advantages and disadvantages in respect to accuracy, measuring time and operating expenses.
The parameters of the used components also influence the measurement result.

4 Literature

[1] Electronic Noise and low Noise design, P. J. Fish, The Macmillan Presse LTD
ISBN 0-333-57309-9 (hardcover)
[2] Electronic Noise and low Noise design, P. J. Fish, The Macmillan Presse LTD
[3] Electronic Noise, Robert King, Chapman And Hall LTD
[4] Electrical Noise, W.R. Bennet, McGraw-Hill Book Company, Inc.